Multi-photon absorption spectra involving the magneto-optical and Wannier - Stark effects in superlattices

1996 J. Phys.: Condens. Matter 8 877

(http://iopscience.iop.org/0953-8984/8/7/012)

J. Phys.: Condens. Matter **8** (1996) 877–887. Printed in the UK

Multi-photon absorption spectra involving the magneto-optical and Wannier–Stark effects in superlattices

B S Monozon†, J L Dunn‡ and C A Bates‡

† Physics Department, Marine Technical University, 3 Lotsmanskaya Street, St Petersburg, 190008, Russia

‡ Physics Department, University of Nottingham, University Park, Nottingham NG7 2RD, UK

Received 15 August 1995, in final form 7 November 1995

Abstract. The interband multi-photon absorption spectrum from a semiconductor superlattice induced by an intense optical wave in the presence of uniform magnetic and electric fields is analysed theoretically. The oscillating electric field of the optical wave and the uniform electric and magnetic fields are all directed perpendicular to the heteroplanes. The explicit dependences of the coefficient of the multi-photon absorption on the frequency and magnitude of the oscillating electric field, on the superlattice parameters and on the magnitudes of the uniform electric and magnetic fields are obtained. The uniform electric field is assumed to be sufficiently strong to provide Wannier–Stark localization of the carriers. The electron–hole cyclotron frequency and the frequency of the Stark oscillations are taken to be commensurate with their ratio being rational. It is shown that, under these conditions and in contrast to one-photon spectra, an enhancement of the series of resonances could arise. It is also shown that the form of the spectrum depends on the relationship between the cyclotron and Stark frequencies and on the number of photons involved.

1. Introduction

During the last decade, the properties of semiconductor superlattices (SL) has attracted much attention. One typical example is the heterostructure formed by alternating layers of GaAs and AlAs semiconductors. The energy spectrum of an electron associated with the Oz-direction (the SL direction) normal to the heterolayers splits into an alternating series of allowed and forbidden minibands. This miniband spectrum is superimposed on the two-dimensional energy-band spectrum associated with the motion of the electrons in the heteroplanes. The unique properties of such superlattices are caused by the combination of both the localized and extended carrier states due to tunnelling through the barriers separating the quantum wells.

The optical response of SLs reflects these unique properties. This is especially the case with optical experiments involving interband optical absorption in the presence of external uniform electric ($\boldsymbol{E}$) and magnetic ($\boldsymbol{B}$) fields. With $\boldsymbol{E}$ parallel to Oz, the finite quasi-classical motion of an electron of charge e along Oz has a frequency $\omega_E = eEa/\hbar$ where a is the period of the SL [1, 2]. This leads to a discrete energy spectrum consisting of equidistant Stark levels separated by an amount $\hbar\omega_E$ whilst the electron can be localized within one period by the available strong electric field. The influence of the Wannier–Stark quantization on the optical response of a superlattice has been investigated previously both theoretically [3–5] and experimentally [6–9]. We note that the localized electron states have a quasi-2D character.

0953-8984/96/070877+11$19.50

The uniform magnetic field $\boldsymbol{B}$ quantizes the electron motion to the heteroplanes. The relevant energy spectrum consists of a sequence of equidistant discrete Landau levels separated by an amount $\hbar\omega_B$ where $\omega_B = eB/m$ and m is the in-plane effective mass. The interband optical absorption spectrum consists of a series of continuous absorption bands associated with the Landau levels [10, 11]. In this case, we have to deal with the quasi-1D states. Of interest is the particular geometry when the applied magnetic and electric fields are parallel to Oz. As the magnetic and electric fields localize the electrons within and perpendicular to the heteroplanes respectively, the dimensions of the electron orbits tend to zero. The overall energy spectrum is discrete and the superlattice optical response consists of δ-function-type resonances which are favourable for experimental observation.

The study of interband magneto-electro-absorption in superlattices commenced about five years ago [12]. Of particular interest is a remarkable effect first observed by Claro *et al* [13]. Under the condition of commensurability between the Stark ω_E and cyclotron ω_B frequencies, they predicted a drastic enhancement of the absorption spectrum. The reason for this is that the Landau levels associated with different Stark levels cross. However, if the Coulomb interaction between the electrons and holes and the mixing between light- and heavy-hole states from the valence band are taken into account, different Landau levels are shown to anticross [14, 15]. The experimental data [14] and theoretical results [15] are given for the Faraday configuration of the optical wave.

It should be noted that most of the previous theoretical papers on this topic are based on either numerical- or variational-type calculations of one-photon effects induced by a weak optical wave. The numerical character of these calculations is a consequence of using a real superlattice potential which consists of a large number of rectangular wells separated by barriers of finite width and height. However, an analytical approach to the calculation of the interband optical transitions in the superlattice has been developed previously [13, 16, 17] in which the superlattice potential barriers are modelled by barriers which are δ-type functions [16, 17]. With this model, explicit analytical expressions for the coefficient of one-photon electro- [16], magneto- [17] and electro-magneto-absorption [13] were obtained.

Very recently, much attention has focused on multi-photon effects in semiconductor superlattices induced by an intense optical wave [18]. Under these circumstances, the non-linear response gives additional information about the properties of the heterostructures. In particular, it was noted [14] that the enhancement of the electro-magneto-absorption of a weak wave, which is caused by the crossing of Landau levels associated with the different inter-well Stark electron–hole transitions [13], could not be tested. This is because, under the localization condition, only vertical inter-well one-photon Stark transitions can occur. Meanwhile, an analytical approach developed originally in [16, 17] has been extended to multi-photon effects. Thus explicit expressions for the coefficient of the multi-photon interband magneto-absorption [19], 2D exciton absorption [20] and electro-absorption [21] have been obtained.

The aim of this paper is to extend the approach given above in [19–21] to the calculation of interband multi-photon absorption in a semiconductor superlattice subjected to both external uniform electric and magnetic fields. The oscillating electric field of the intense optical wave and the uniform electric and magnetic fields are all directed along Oz. The effective mass approximation and quasi-energetic approach will be used. The superlattice is again modelled by a limiting form of the Kronig–Penney potential consisting of a periodic chain of δ-function-type barriers. The explicit dependences of the coefficient of the multiphoton absorption on the frequency and magnitude of the oscillating electric field, on the superlattice parameters and on the magnitudes of the uniform electric and magnetic fields will be obtained. In particular, it should be emphasized that here we concentrate on

the multi-photon component in the absorption spectra caused by the intense optical wave. The exciton interaction, the mixing between light- and heavy-hole states and the effects of different superlattice sub-bands are excluded on account of the complexities which they introduce into the analysis.

Our main concern is the magnitude of the uniform electric field and the relationship between the cyclotron ω_B and Stark ω_E frequencies. The uniform electric field is regarded as having sufficient strength to provide Wannier–Stark localization of the carriers. The electron–hole cyclotron frequency and the frequency of the Stark oscillations are taken to be commensurate such that their ratio is rational. It will be shown that the last condition leads to an enhancement of the series of discrete resonances within the multi-photon absorption spectrum. This is due to the fact that, in contrast to the one-photon spectrum, intense non-vertical multi-photon Stark transitions occur. Thus the predictions given in [13] could be tested in principle. The localization condition simplifies the absorption spectrum compared to the case of a weak uniform electric field. The form of the multi-photon absorption spectrum depends upon the number of photons involved.

2. Quasi-energetic states of the carriers

Let us consider an electron in a semiconductor superlattice which consists of a large number N' of periods a, in the presence of an oscillating electric field $\boldsymbol{\eta} F_0 \cos \omega t$ of angular frequency ω, magnitude F_0, polarization (unit) vector $\boldsymbol{\eta}$, an external uniform electric field $\boldsymbol{E}$ and an uniform magnetic field $\boldsymbol{B}$. Assuming that the external fields, including the superlattice potential, satisfy the effective mass approximation, the equation for the envelope wavefunction Ψ describing a particle at a position $\boldsymbol{r}$ in a simple band with effective mass m is given by

$$\left\{ \frac{1}{2m}(-\mathrm{i}\bar{h}\nabla - \tfrac{1}{2}e[\boldsymbol{B}\cdot\boldsymbol{r}])^2 + V(z) - (\boldsymbol{E} + \boldsymbol{\eta} F_0 \cos\omega t)\cdot\boldsymbol{r} \right\} \Psi(\boldsymbol{r}, t) = \mathrm{i}\bar{h}\frac{\partial \Psi(\boldsymbol{r}, t)}{\partial t} \tag{2.1}$$

where the periodic superlattice potential formed by the δ-type barriers of power α_0 may be written in the form

$$V(z) = \alpha_0 \sum_s \delta(z - as) \text{ with } V(z) = V(z + a) \text{ and } \alpha_0 > 0 \ (s\text{-integral}).$$

If the electric and magnetic fields are directed along the SL direction such that $\boldsymbol{E}$, $\boldsymbol{\eta}$, $\boldsymbol{B}$ and Oz are all parallel, the solution to equation (2.1) is

$$\Psi(\boldsymbol{r}, t)\mathrm{e}^{-\mathcal{E}_\perp t/\bar{h}}\Phi_\perp(\boldsymbol{\rho})\varphi(z, t) \tag{2.2}$$

where $\Phi_\perp(\boldsymbol{\rho})$ is the transverse wavefunction of an electron with energy $\mathcal{E}_\perp$ in the uniform magnetic field $\boldsymbol{B}$ and $\varphi(z, t)$ obeys the equation

$$-\frac{\bar{h}^2}{2m}\frac{\partial^2\varphi}{\partial z^2} + [V(z) - (E + F_0\cos\omega t)]\varphi = \mathrm{i}\bar{h}\frac{\partial\varphi}{\partial t}. \tag{2.3}$$

Equation (2.3) has been studied in detail in [21] from which we obtain

$$\varphi(z, t) = \frac{a\sqrt{N'}}{2\pi}\int_{-\pi/a}^{+\pi/a} \psi[z, q(t, k)] \exp\left\{ -\frac{\mathrm{i}}{\bar{h}}\int_0^t (\mathcal{E} - \bar{\epsilon}(k) + \epsilon[q(\tau, k)])\,\mathrm{d}\tau \right.$$
$$\left. -\frac{\mathrm{i}}{eE}\int_0^k (\bar{\epsilon}(k') - \mathcal{E} + \tfrac{1}{2}eEa)\,\mathrm{d}k' \right\} \tag{2.4}$$

where $\epsilon(k)$ is the energy of the ground allowed miniband given by

$$\epsilon(k) = b + \tfrac{1}{2}\Delta(1 - \cos ka) \tag{2.5}$$

with the lower boundary at $\epsilon(0) = b$ and of width Δ, and where $\psi(z, k)$ is the ground miniband Bloch function of the electron in the superlattice having an average momentum $\overline{h}k$. Also

$$q(t, k) = k + \frac{eF_0}{\overline{h}\omega} \sin \omega t \qquad \bar{\epsilon}(k) = \frac{1}{T} \int_0^T \epsilon[q(\tau, k)] \, \mathrm{d}\tau \qquad \text{with } T = \frac{2\pi}{\omega}. \tag{2.6}$$

Explicit expressions for the miniband parameters b, Δ and the Bloch functions ψ defined by the effective mass m, the period of the superlattice a and with barrier power α_0 are given in [16, 17, 19–21] for the case of weak barrier penetration.

It is easy to see that the function (2.4) satisfies the conditions

$$\varphi(z, t) = \mathrm{e}^{-\mathrm{i}\mathcal{E}t/\overline{h}} f(z, t) \text{ such that } f(z, t + T) = f(z, t) \text{ and } \varphi(z, t + T) = \mathrm{e}^{-\mathrm{i}\mathcal{E}T/\overline{h}} \varphi(z, t). \tag{2.7}$$

This means that the expression (2.4) is the quasi-energetic wave function for the Stark energy levels [21] with quasi-energy $\mathcal{E}_\sigma$ [22] given by

$$\mathcal{E}_\sigma = eEa(\sigma + \tfrac{1}{2}) + \frac{a}{2\pi} \int_0^{2\pi/a} \bar{\epsilon}(k) \, \mathrm{d}k \qquad \text{where } \sigma = 0, \pm 1, \pm 2, \pm 3, \ldots. \tag{2.8}$$

The expression (2.4) is valid under the condition

$$\nu = \frac{eEa}{\overline{h}\omega} \ll 1. \tag{2.9}$$

For example, if $\overline{h}\omega \approx 1$ eV, $a \approx 50$ Å and $E \approx 10^7$ V m^{-1}, then $\nu \approx 5 \times 10^{-2}$. Taking into account the periodicity of the Bloch function (namely $\psi(z, k) = \psi(z, k + 2\pi/a)$), it may be expanded in a Fourier series in the form

$$\psi(z, q) = \frac{1}{\sqrt{N'}} \sum_\mu Q_\mu(z) \, \mathrm{e}^{\mathrm{i}qa\mu}. \tag{2.10}$$

To obtain the function (2.4) and the quasi-energy (2.8) in explicit forms, we make the assumption that, in the case of a real superlattice and in the presence of a real oscillating field F_0, the parameter β is such that

$$\beta = \frac{eF_0 a}{\overline{h}\omega} \ll 1. \tag{2.11}$$

Thus if $\overline{h}\omega \approx 1$ eV and $a \approx 50$ Å as above and also $F_0 \approx 5 \times 10^7$ V m^{-1}, then $\beta \approx 0.25$.

Under conditions (2.9) and (2.11) and using the expressions (2.4) for the wavefunctions, then (2.7) and the quasi-energies (2.8) become [21]

$$f_\sigma(z, t) = \sum_n Q_{n-\sigma}(z)(-1)^n J_n(\chi) \qquad \text{and} \qquad E_\sigma = eEa(\sigma + \tfrac{1}{2}) + b + \tfrac{1}{2}\Delta \tag{2.12}$$

where $J_n(\chi)$ are Bessel functions with

$$\chi = \frac{\zeta}{2\nu}(1 - \tfrac{1}{4}\beta^2) = \frac{\Delta}{2eEa}(1 - \tfrac{1}{4}\beta^2) \tag{2.13}$$

and where

$$\zeta = \frac{\Delta}{\overline{h}\omega} \ll 1.$$

Since $\Delta \sim 0.1$ eV, we have $\zeta \sim 0.1$.

It follows that, from the function given in (2.12), the degree of the localization of the electrons is defined by the Bessel function $J_n(\chi)$. If $\chi \ll 1$, then $J_0 = 1$ and $J_n \ll 1$ ($n \neq 0$) and the electron having an energy $\mathcal{E}_\sigma$ is localized in the superlattice cell with index $s = -\sigma$. The expression (2.13) for χ shows that localization increases with an increase in the magnitudes of both the uniform $\boldsymbol{E}$ and oscillating F_0 electric fields.

3. Coefficient of the interband multi-photon absorption

The outline of the derivation of the multi-photon absorption coefficient will be given below but further details of the calculation can be found in [21]. We consider the interband absorption as a transition of the electron–hole pair from the ground state to an excited state in which an electron (e) is initially in the conduction ground miniband and a hole (h) is initially in the valence miniband. The interband optical absorption essentially depends upon the kinetic parameter $\Omega_{\mathrm{eh},l}\tau$ where τ is the relaxation time of the electron momentum in the conduction band and $\Omega_{\mathrm{eh},l}$ is the frequency of the l-photon transition. This frequency can be estimated from the approximation $\Omega_{\mathrm{eh},l} \sim \omega/\gamma^l$ where ω is the frequency of light as before and γ ($\gg 1$) is the parameter defined in (3.6) below at $\Theta = 0$. Approximate estimates for the parameters for the GaAs/GaAlAs superlattice system (namely $\mathcal{E}_g \sim 1.5$ eV, $a \sim 50$ Å, $(\Delta_\mathrm{e} + \Delta_\mathrm{h}) \sim 0.1$ eV and $\tau = 10^{-11}$ s) may be made. With an electric field $F_0 \sim 10^7$ V m^{-1}, the kinetic parameter is such that, for three-photon absorption ($l = 3$, $3\bar{h}\omega \sim \mathcal{E}_g$) $\Omega_{\mathrm{eh},l}\tau \sim 0.02$. Under the condition $\Omega_{\mathrm{eh},l}\tau \ll 1$ which is anticipated, the photo-generated electrons do not accumulate in the conduction band due to relaxation processes, which in turn leads to the interband absorption. Thus the one-band assumption becomes valid. The ground state is then described by the function $\Psi_0(\boldsymbol{r}_\mathrm{e}, \boldsymbol{r}_\mathrm{h}) = \delta(\boldsymbol{r}_\mathrm{e} - \boldsymbol{r}_\mathrm{h})$ and the excited state by the function

$$\Psi(\boldsymbol{r}_\mathrm{e}, \boldsymbol{r}_\mathrm{h}, t) = \Psi_\mathrm{e}(\boldsymbol{r}_\mathrm{e}, t)\Psi_\mathrm{h}(\boldsymbol{r}_\mathrm{h}, t). \tag{3.1}$$

The electron function $\Psi_\mathrm{e}(\boldsymbol{r}_\mathrm{e}, t)$ is defined by expressions (2.2), (2.7), (2.12) and (2.13) with the addition of subscripts 'e' to all parameters. Similarly, the hole function $\Psi_\mathrm{h}(\boldsymbol{r}, t)$ can be obtained from the electron function by replacing the subscript 'e' by the subscript 'h', σ by σ', t by $-t$, e by $-e$ and taking complex conjugates. Also, in the expression (2.12), the electron Wannier function $Q_\mu(z_\mathrm{e})$ should be replaced by the hole function $\bar{Q}^*_{\mu'}/(z_\mathrm{h})$ with $\bar{Q}_\mu = Q_{-\mu}$.

The coefficient of the interband dipole transition under the oscillating electric field is defined by the matrix element of the operator

$$P(t) = \frac{\mathrm{i}\bar{h}eF_0 p_{\mathrm{ehz}}}{m_0\mathcal{E}_g}\cos\omega t \tag{3.2}$$

where p_{ehz} is the matrix element of the momentum operator between the amplitudes of the Bloch functions of the electron and hole bands separated by the forbidden gap $\mathcal{E}_g$. The matrix element of the operator $P(t)$ given in (3.2) between the wavefunctions Ψ_0 and Ψ given in (3.1) defines the transition rate. This, in turn, defines the coefficient of absorption α in the general form [21]

$$\alpha = \sum_l \alpha_l$$

where

$$\alpha_l = \frac{2\pi\omega\bar{h}^2e^2|p_{\mathrm{ehz}}|^2}{\epsilon_0 cVm_0^2n_0\mathcal{E}_g^2}\sum_{\mathrm{e,h}}|A_l(\omega)|^2\delta(l\bar{h}\omega - \mathcal{E}_g - \mathcal{E}_{\perp\mathrm{e}} - \mathcal{E}_{\perp\mathrm{h}} - \mathcal{E}_\sigma - \mathcal{E}_{\sigma'}) \tag{3.3}$$

where α_l is the coefficient of l-photon interband absorption, $n_0(\omega)$ is the refractive index, c is the speed of light, $V = L_xL_yL_z$ is the volume of crystal, $\Sigma_{\mathrm{e,h}}$ is a sum over band states and $A_l(\omega)$ is the Fourier coefficient:

$$A_l(\omega) = \frac{\omega}{2\pi}\int_{-\pi/\omega}^{+\pi/\omega}\mathrm{e}^{\mathrm{i}l\omega}\cos\omega t\int\delta(\boldsymbol{r}_\mathrm{e} - \boldsymbol{r}_\mathrm{h})\Phi^*_{\perp\mathrm{e}}(\boldsymbol{\rho}_\mathrm{e})\Phi^*_{\perp\mathrm{h}}(\boldsymbol{\rho}_\mathrm{h})f^*_\mathrm{e}(z_\mathrm{e}, t)f^*_\mathrm{h}(z_\mathrm{h}, t)\,\mathrm{d}\boldsymbol{r}_\mathrm{e}\,\mathrm{d}\boldsymbol{r}_\mathrm{h}\,\mathrm{d}t. \tag{3.4}$$

Using the integral representation for Bessel functions in the expression (2.12) and then substituting (2.12) into (3.4), the form for the coefficient $A_l(\omega)$ becomes

$$A_l(\omega) = \sqrt{L_x L_y}\Phi_\perp(0)\frac{1}{2\pi}\int_{-\pi}^{+\pi} \exp\left\{\mathrm{i}\left[(\sigma-\sigma')\Theta - \frac{\zeta_{\mathrm{eh}}}{2\nu}(1-\tfrac{1}{4}\beta^2)\sin\Theta\right]\right\} B_l(\Theta)\,\mathrm{d}\Theta \tag{3.5}$$

where $\Phi_\perp(\boldsymbol{\rho})$ and $\boldsymbol{\rho} = \boldsymbol{\rho}_\mathrm{e} - \boldsymbol{\rho}_\mathrm{h}$ is a function of the relative transverse motion

$$B_l(\omega,\Theta) = \frac{1}{2\sqrt{l\pi}}\mathrm{e}^{-\mathrm{i}\frac{1}{2}\pi(l-1)+\frac{1}{2}l}\left(\frac{1}{2\gamma}\right)^{l-1}\sin[-2l\xi+\tfrac{1}{2}l\pi] \tag{3.6}$$

and where

$$2\xi(\Theta) = \left(\frac{\zeta_{\mathrm{eh}}}{l}\right)^{1/2}\frac{\sin\Theta}{(\cos\Theta)^{1/2}} \qquad \text{and} \qquad \frac{1}{\gamma^2(\Theta)} = \frac{\zeta_{\mathrm{eh}}}{4l}\beta^2\cos\Theta$$

such that $\zeta_{\mathrm{eh}} = \zeta_\mathrm{e} + \zeta_\mathrm{h} \ll 1$ and $\gamma \gg 1$ with ζ_{eh} given in (2.13) in which Δ is the width of the relevant electron or hole miniband.

The expressions (3.3), (3.5) and (3.6) define the coefficient of the l-photon interband electro-absorption in the superlattice. The above expressions have a common character but they are not related to any specific transverse states.

4. Results and discussion

4.1. General results

The wavefunctions $\Phi_\perp(\boldsymbol{\rho})$ and energies $\mathcal{E}_{\perp\mathrm{e,h}}$ for the Landau levels of the electron–hole pair in the presence of a magnetic field $\boldsymbol{B}$ directed along Oz have the well known form [13, 16]

$$\Phi_\perp(0) = \frac{1}{\sqrt{2\pi a_B}} \qquad \mathcal{E}_{\perp\mathrm{e}} + \mathcal{E}_{\perp\mathrm{h}} = \mathcal{E}_{\perp N} = \hbar\omega_B(N+\tfrac{1}{2}) \pm (\beta_\mathrm{e}+\beta_\mathrm{h})B$$
$$(N = 0, 1, 2, \ldots) \tag{4.1}$$

where

$$a_B = \sqrt{\frac{\hbar}{eB}} \qquad \omega_B = \frac{eB}{\mu} \qquad \text{and} \qquad \mu^{-1} = m_\mathrm{e}^{-1} + m_\mathrm{h}^{-1}$$

and where $\beta_{\mathrm{e,h}}$ are the effective Bohr magnetons. The longitudinal electron–hole energy (2.13) is given by

$$\mathcal{E}_\sigma + \mathcal{E}_{\sigma'} = \hbar\omega_E\delta + b_\mathrm{e} + b_\mathrm{h} + \tfrac{1}{2}\Delta_\mathrm{e} + \tfrac{1}{2}\Delta_\mathrm{h} \tag{4.2}$$

where

$$\omega_E = \frac{eEa}{\hbar} \qquad \text{and} \qquad \delta = \sigma - \sigma' = 0, \pm1, \pm2, \ldots.$$

The sum in (3.3) is of the form

$$\sum_{\mathrm{e,h}} = \sum_{N,\sigma,\sigma'} = N'\sum_{N,\delta}. \tag{4.3}$$

On substituting the expression (3.6) into (3.5), the coefficient $A_l(\omega)$ can be expressed in terms of Bessel functions. Substituting for $\Phi_\perp(0)$ and $(\mathcal{E}_{\perp\mathrm{e}} + \mathcal{E}_{\perp\mathrm{h}})$ from (4.1), and for $(\mathcal{E}_\sigma + \mathcal{E}_{\sigma'})$ from (4.2) into (3.3) and using the summation rules (4.3), the result for α_l is

$$\alpha_l(\omega) = \alpha_0\frac{e^l}{4l\pi}\left(\frac{\zeta_{\mathrm{eh}}\beta^2}{16l}\right)^{l-1}\Lambda_l(\omega) \tag{4.4}$$

where

$$\alpha_0 = \frac{\hbar^2\omega e^2|p_{\mathrm{ehz}}|^2}{4\pi^2 a_B^2\epsilon_0 cam_0^2 n_0\mathcal{E}_g^2}$$

and where

$$\Lambda_l(\omega) = \sum_{N=0}^{\infty} \sum_{\delta=-\infty}^{+\infty} |M_\delta^{(l)}|^2 \delta(l\bar{h}\omega - \bar{\mathcal{E}}_g - \bar{h}\omega_E\delta - \bar{h}\omega_B N) \tag{4.5}$$

with

$$\bar{\mathcal{E}}_g = \mathcal{E}_g + b_e + b_h + \tfrac{1}{2}(\Delta_e + \Delta_h) + \tfrac{1}{2}\omega_B \pm (\beta_e \pm \beta_h)B.$$

The matrix elements are as follows [21]:

(a) for the transitions involving an odd number of photons, such that $l = 2s + 1$, with $s = 0, 1, 2, \ldots$ we have

$$M_\delta^{(l)}(\nu, \beta) = \frac{1}{2^s} \sum_{j=0}^{s} \binom{s}{j} J_{s-\delta-2j}(\chi_{eh}) \tag{4.6}$$

(b) for the transitions involving an even number of photons, such that $l = 2(s+1)$ with $s = 0, 1, 2, \ldots$ we have

$$M_\delta^{(l)}(\nu, \beta) = (l\zeta_{eh})^{1/2} \frac{1}{2^{s+1}} \sum_{j=0}^{s} \binom{s}{j} [J_{s+1-\delta-2j}(\chi_{eh}) - J_{s-1-\delta-2j}(\chi_{eh})] \tag{4.7}$$

where

$$\chi_{eh} = \frac{\zeta_{eh}}{2\nu}(1 - \tfrac{1}{4}\beta^2) \text{ with } \zeta_{eh}, \nu \text{ and } \beta \ll 1 \text{ and with } \binom{s}{j} = \frac{s!}{j!(s-j)!}$$

and where the latter are the binomial coefficients.

The expressions (4.4)–(4.7) define the coefficient of the l-photon interband absorption in the presence of the uniform electric and magnetic fields parallel to the SL direction. With a weak oscillating electric field F_0 (i.e. $\beta \to 0$), only the one-photon ($l = 1$, $s = 0$) absorption (4.4) is large. In this case, the matrix element (4.6) becomes $M_\delta^{(1)} = J_{-\delta}(\chi_{eh})$, and the coefficient α_l from (4.4) tends to the previous results for the one-photon absorption [13, 16].

From equations (4.4) and (4.5), we see that the l-photon absorption spectrum consists of a superposition of the Stark ladders formed by the δ-function-type resonances labelled by the index δ. Their oscillator strengths are governed by the matrix elements $M_\delta^{(l)}$. The ladder formed by the Landau levels labelled by the index N is associated with each Stark resonance.

The matrix elements $M_\delta^{(l)}$ consist of Bessel functions. Their argument depends upon the value of the factor ζ_{eh}/ν. For arbitrary values of $\zeta_{eh}/\nu \geqslant 1$, the transitions with different δ contribute almost equally to the absorption spectrum (4.5). Interband transitions between any pair of Stark levels ($\sigma \to \sigma'$) and between any wells are possible. In this case, the multi-photon absorption spectrum consists of the complex superposition of the Stark ladders, each of which consists of a large number of resonances of nearly equally intensity. These interband transitions between the extended states of carriers are the essence of the multi-photon magneto-optical Franz–Keldysh effect in the superlattice [23]. Unfortunately, it is unrealistic to investigate the details of this spectrum experimentally.

Let us consider the most attractive case for the localization of carriers caused by an uniform electric field $\boldsymbol{E}$ of sufficient strength such that $\zeta_{eh}/\nu \ll 1$. With such fields, the magnitude of the argument of the Bessel functions in equations (4.6) and (4.7) becomes $\chi \ll 1$. Note also that an increase in the magnitude of the oscillating electric field $F_0 \sim \beta$ causes χ_{eh} to decrease. Under the localization condition, $\chi_{eh} \ll 1$, and thus only the Bessel function $J_0(\chi_{eh}) \approx 1$ contributes to the matrix elements (4.6) and (4.7) as $J_n(\chi_{eh}) \sim \chi_{eh}^n \ll 1$

for $n \neq 0$. As a consequence, interband intense multi-photon transitions between a limited number of Stark levels ($\sigma \to \sigma'$) and between a limited number of wells are possible. It follows from (4.6) and (4.7) that, for the one-photon absorption ($l = 1$, $s = 0$), only the vertical interwell transitions with $\delta = 0$ ($\sigma \to \sigma'$) are allowed. For two- ($l = 2$, $s = 0$) and three-photon absorption ($l = 3$, $s = 1$), non-vertical transitions with $\delta = \pm 1$ are possible. As a result, under the localization condition the multi-photon absorption spectrum will be formed by the Stark ladders each of which consists of a limited number (one for $l = 1$, two for $l = 2, 3, \ldots$ etc) of intense resonances. Optical transitions occurring between the localized states of a single quantum well cause the multi-photon magneto-optical Wannier–Stark effect. It is apparent that such an experimental spectrum is more readily available for study.

4.2. Commensurate Stark and cyclotron frequencies

Let us consider the most attractive case when the Stark (ω_E) and cyclotron (ω_B) frequencies are taken to be commensurate. This means that $\omega_B/\omega_E = p/q$ where p and q are integers having no common divisors. Under this condition, certain of the Landau levels associated with different Stark levels coincide and an enhancement of the relevant resonances occur. Under this commensurate condition, expression (4.5) becomes

$$\Lambda_l(\omega) = \frac{q}{\hbar\omega_E} \sum_{N=0}^{\infty} \sum_{\delta=-\infty}^{+\infty} |M_\delta^{(l)}|^2 \delta\left(q\frac{l\hbar\omega - \bar{\mathcal{E}}_g}{\hbar\omega_E} - (q\delta + pN) \right). \tag{4.8}$$

From (4.8), it follows that the absorption spectrum consists of δ-function-type resonances with frequencies ω_n given by

$$\frac{l\hbar\omega_n - \bar{\mathcal{E}}_g}{\hbar\omega_E} = \frac{n}{q} \qquad \text{where } n = q\delta + pN. \tag{4.9}$$

The intensity of the resonance positioned at ω_n is defined by the sum of the intensities $|M_\delta^{(l)}|^2$ with different indexes δ and N corresponding to the fixed number n. Under the localization condition, the allowed values of δ are limited by the number l of photons involved. As a result, and in contrast to the case of a weak uniform electric field E [13], the distance between the neighbouring values of n is not universally equal to one unit and the resonances are not equally spaced but have a separation of $\hbar\omega_E/q$.

4.3. The form of the spectrum

We consider in turn the most commonly studied experimental absorption spectra involving one, two and three photons.

4.3.1. One-photon absorption (l =1, s =0). As the transitions with $\delta = 0$ are allowed, there is no superposition of the Landau ladders associated with different Stark transitions. As noted in [14] under the localization condition, the predicted enhancement of the one-photon absorption of the weak optical wave generated by the crossing Landau levels [13] cannot be tested.

4.3.2. Two- (l = 2, s = 0) and three-photon (l = 3, s = 1) absorption. For these numbers of photons, the transitions with $\delta = \pm 1$ are possible. From (4.9), it follows that, for two Landau levels associated with the Stark transitions for which $\delta = \pm 1$ and which cross, their indices $N_{+,-}$ must satisfy the condition

$$n = q + pN_+ = -q + pN_- \qquad \text{with } N_{+,-} = 0, 1, 2, \ldots. \tag{4.10}$$

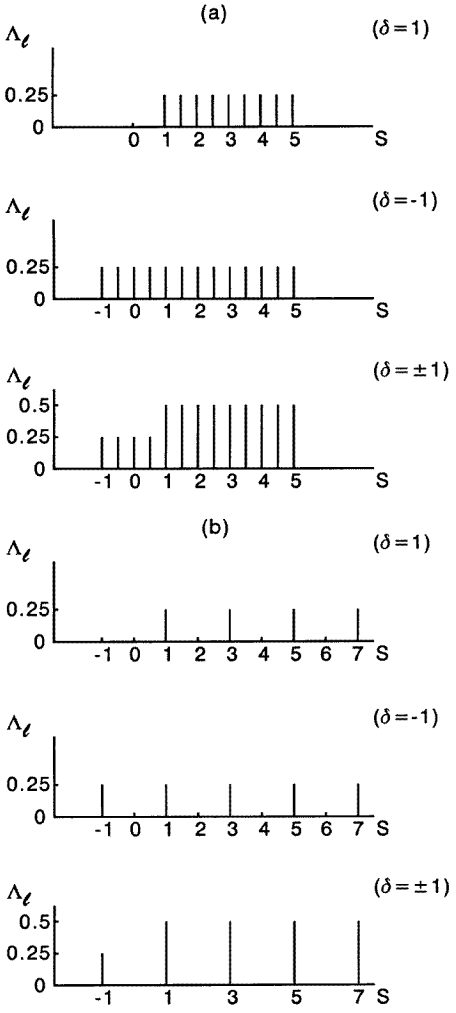

Figure 1. The relative oscillator strengths Λ_l of the two- ($l = 2$) and three-photon ($l = 3$) separate magneto-electro-absorption spectra associated with the Stark transitions $\delta = +1$ and $\delta = -1$ and the superimposed spectra ($\delta = \pm 1$) versus

$$s = \frac{l\bar{h}\omega_n - \bar{\mathcal{E}}_g}{\bar{h}\omega_E}$$

for (a) $\omega_B/\omega_E = 1/2$ and (b) $\omega_B/\omega_E = 2/1$.

From (4.8), the oscillator strength of the resonance positioned at ω_n is proportional to $\Lambda_l(\omega_n)$ which is given by

$$\Lambda_l(\omega_n) \sim \sum_{N_{+,-}=0}^{\infty} \sum_{\delta=\pm 1} |M_\delta^{(l)}|^2 \tag{4.11}$$

where

$$M_{\pm 1}^{(3)} = (2\zeta_{\text{eh}})^{-1/2} M_{\pm 1}^{(2)} = \tfrac{1}{2}. \tag{4.12}$$

The intensities of the crossed enhanced resonances are doubled compared to those seen in the single spectra.

Using the expressions (4.10), we obtain

$$N_+ - N_- = 2q/p \tag{4.13}$$

for the relationship between N_+ and N_-. As $(N_+ - N_-)$ is an integer, then $p = 1, 2$. Moreover, if $p = 1$ and $q = 2$, then enhanced resonances occur at positions ω_n for $n = 2, 3, 4, \ldots$ whereas the four normal resonances are defined by the indices $-2 \leqslant n \leqslant 1$. Alternatively, if $p = 2$ and $q = 1$, then enhanced resonances occur at ω_n with $n = 1, 3, 5, \ldots$ whereas only the one resonance at the position ω_{-1} is not enhanced. The shapes of the relevant electro-magneto-absorption spectra, which are the same for $l = 2$ and $l = 3$, are depicted in figure 1.

In contrast to the one-photon absorption of the weak wave, the predicted enhancement of the absorption [13] could be detected in principle in the multi-photon magneto-optical Wannier–Stark spectra. As the electron–hole interaction and valence-band mixing prevent the crossing of the levels [14, 15], then superlattices without these effects are favoured experimentally. In the case of a marked exciton contribution, the infrared shift of the resonances can come into being and 'crossing' turns into 'anticrossing' [24]. The anticrossing of the Wannier–Stark interacting states associated with different superlattice sub-bands must always be kept in mind [25]. The reason is that the 'anticrossing' mentioned above occurs in cases of both a Faraday [14, 15] and a Voigt geometry (for $\boldsymbol{\eta}$ parallel to Oz) for the optical wave considered here.

On considering possible experiments, estimates of suitable values for the parameters for the GaAs/$Ga_{1-x}Al_xAs$ superlattice system (with $x = 0.35$) may be made for the case when the complex valence band formed by the subbands associated with light (l) and heavy (h) holes. For such a structure, the total width of the minibands is given by $(\Delta_e + \Delta_{lh}) = 0.14$ eV and $(\Delta_e + \Delta_{hh}) = 0.094$ eV [21]. With an electric field $E = 5 \times 10^7$ V m^{-1}, the parameter χ_{eh} is such that, for electrons and heavy holes, $\chi_{e,hh} = 0.941$ and for electrons and light holes $\chi_{e,lh} = 1.42$. With such an electric field, the heavy holes are almost totally localized. By increasing the electric field E, first the light holes and then the electrons will become localized as the localization condition $\chi_{eh} \ll 1$ is reached in the experiment.

5. Conclusion

In summary, we have developed an analytical approach to the problem of calculating of the multi-photon interband absorption spectrum in the presence of uniform electric and magnetic fields which are directed parallel to the oscillating electric field of the optical wave and the superlattice axis. The uniform electric field is regarded as having sufficient strength so that the carriers are localized. The electron–hole cyclotron frequency and frequency of the Stark oscillations are taken to be commensurate. It has been shown that, under these conditions and in contrast to the one-photon spectrum, an enhancement of the series of the resonances occurs in principle. However, it is clear that the form of the spectrum depends on the relationship between the cyclotron and Stark frequencies and on the number of photons involved.

Acknowledgment

One of the authors (BSM) gratefully acknowledges the Royal Society support which has enabled this collaborative programme to proceed.

References

[1] Feuer P 1952 *Phys. Rev.* **88** 92
[2] Wannier G H 1960 *Phys. Rev.* **117** 432
[3] Bleuse J, Bastard G and Voisin P 1988 *Phys. Rev. Lett.* **60** 220
[4] Dignam M M and Sipe J E 1990 *Phys. Lett.* **64** 1797
[5] Dignam M M and Sipe J E 1991 *Phys. Rev.* B **43** 4097
[6] Mendez E E, Agullo-Roeda F and Hong J M 1988 *Phys. Rev. Lett.* **60** 2426
[7] Agullo-Roeda F, Brum J A, Mendez E E and Hong J M 1990 *Phys. Rev.* B **41** 1676
[8] Mendez E E, Agullo-Roeda F and Hong J M 1990 *Appl. Phys. Lett.* **56** 2545
[9] Voisin P and Bleuse J 1990 *Phys. Rev. Lett.* **61** 1639
[10] Maan J C 1988 *Surf. Sci.* **196** 518
[11] Maan J C, Guldner Y, Vieren J P, Voisin P, Voos M, Chang L L and Esaki L 1981 *Solid State Commun.* **39** 683
[12] Ferreira R, Soucail B, Voisin P and Bastard G 1990 *Phys. Rev.* B **42** 1104
[13] Claro F, Pacheco M and Barticevic Z 1990 *Phys. Rev. Lett.* **64** 3058
[14] Alexandrou A, Mendez E E and Hong J M 1991 *Phys. Rev.* B **44** 1934
[15] Pacheco M, Barticevic Z and Claro F 1992 *Phys. Rev.* B **46** 15 200
[16] Zhilich A G 1992 *Sov. Phys.–Solid. State* **34** 1875
[17] Zhilich A G 1994 *Phys. Solid State* **36** 435
[18] Wagh P M, Kubakaddi S S and Mulimani B G 1993 *Semicond. Sci. Technol.* **8** 2072
[19] Monozon B S, Zhilich A G, Dunn J L and Bates C A 1994 *J. Phys.: Condens. Matter* **6** 4009
[20] Monozon B S, Zhilich A G, Dunn J L and Bates C A 1994 *J. Phys.: Condens. Matter* **6** 10 001
[21] Monozon B S, Dunn J L and Bates C A 1994 *Phys. Rev.* B **50** 17 097
[22] Zeldovich Y B 1967 *Sov. Phys.–JETP* **24** 1006
[23] Franz W 1958 *Z. Naturf.* a **13** 484
Keldysh L V 1958 *Sov. Phys.–JETP* **7** 788
[24] Barticevic Z, Pachecho M and Claro F 1995 *Phys. Rev.* B **51** 14 414
[25] Bastard G, Ferreira R, Chelles S and Voisin P 1994 *Phys. Rev.* B **50** 4445